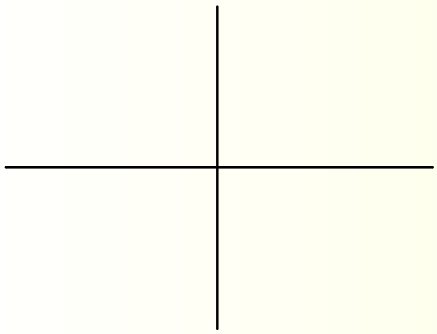
Calculus Warm Up #5-5

Sketch the solid created when the graph of

$$y = \frac{1}{4}(x^2 + 12)$$

is revolved about the x-axis on the interval [-2, 4].

Write the integral and find the volume.

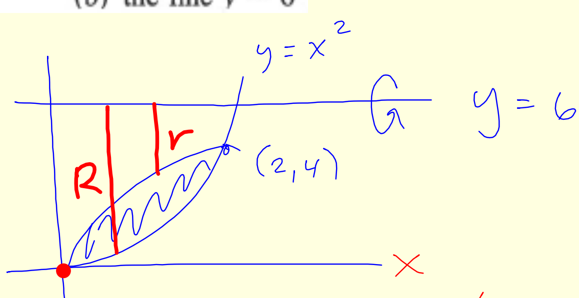


HW Questions: p.310

In Exercises 13–16, find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the indicated lines.

15. $y = x^2$, $y = 4x - x^2$

(a) the x-axis (b) the line $y = 6$



$R = 6 - x^2$ $\qquad$ $r = 6 - (4x - x^2)$

$$V = \pi \int_0^2 \left[(6 - x^2)^2 - (x^2 - 4x + 6)^2 \right] dx$$

In Exercises 17–20, find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the x-axis.

17. $y = x\sqrt{4 - x^2}$, $y = 0$

19. $y = \dfrac{1}{x}$, $y = 0$, $x = 1$, $x = 4$

In Exercises 21–24, find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the line $y = 4$.

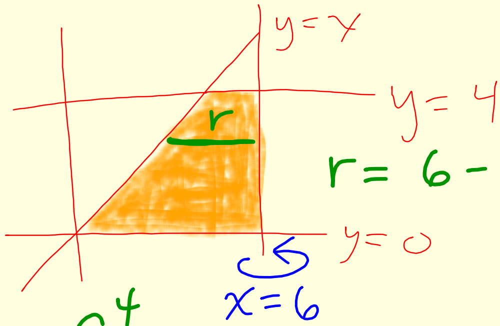
21. $y = x$, $y = 3$, $x = 0$

23. $y = \dfrac{1}{x^2}$, $y = 0$, $x = 1$, $x = 4$

In Exercises 25–28, find the volume of the solid generated by revolving the region bounded by the graphs of the given equations about the line $x = 6$.

25. $y = x$, $y = 0$, $y = 4$, $x = 6$

27. $x = y^2$, $x = 4$



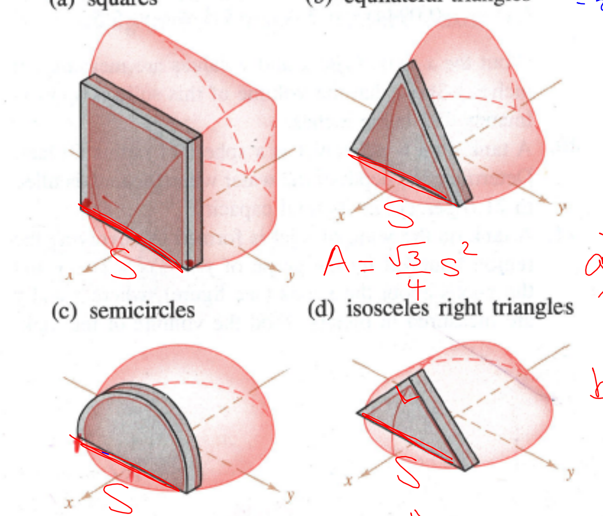
$r = 6 - y$

$$\pi \int_0^4 (6-y)^2 \, dy$$

45. Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 4$, with the indicated cross sections taken perpendicular to the x-axis.

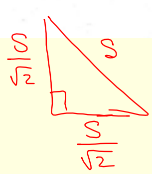
(a) squares (b) equilateral triangles

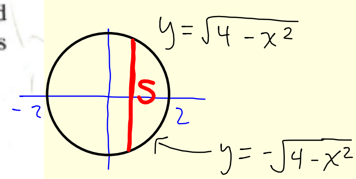
(c) semicircles (d) isosceles right triangles



$A = \frac{\sqrt{3}}{4} s^2$

$A = \frac{1}{2}\pi \left(\frac{S}{2}\right)^2$





$S = \sqrt{4-x^2} - (-\sqrt{4-x^2})$

$\boxed{S = 2\sqrt{4-x^2}}$

a) $V = 2\int_0^2 (2\sqrt{4-x^2})^2 \, dx$

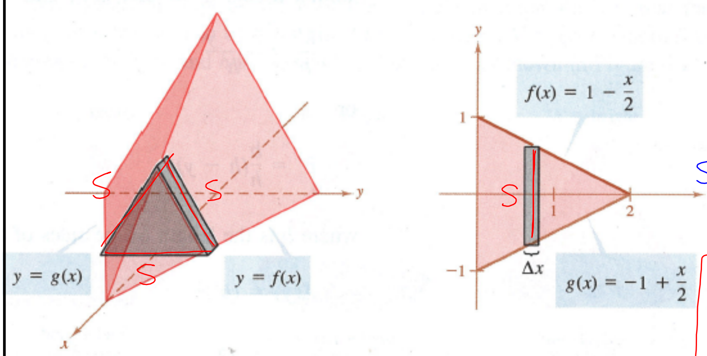
b) $V = 2\int_0^2$

Find the volume of the solid whose base is the area bounded by the lines

$$f(x) = 1 - \frac{x}{2}, \qquad g(x) = -1 + \frac{x}{2}, \qquad \text{and} \qquad x = 0$$

and whose cross sections perpendicular to the x-axis are equilateral triangles,

$A = \frac{\sqrt{3}}{4} s^2$



Triangular base in xy-plane

$s = f(x) - g(x)$

$S = 1 - \frac{x}{2} - \left(-1 + \frac{x}{2}\right)$

$S = 2 - x$

$\boxed{A(x) = \frac{\sqrt{3}}{4}(2-x)^2 \quad \text{area of a cross section}}$

$V = \int A(x)\,dx$

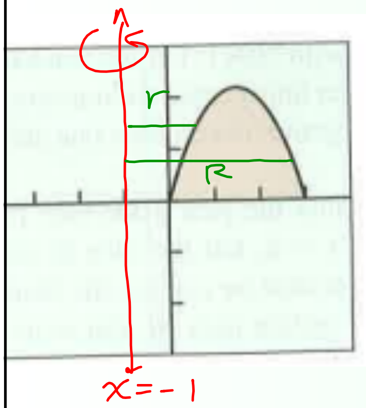
$V = \frac{\sqrt{3}}{4}\int_0^2 (2-x)^2\,dx$

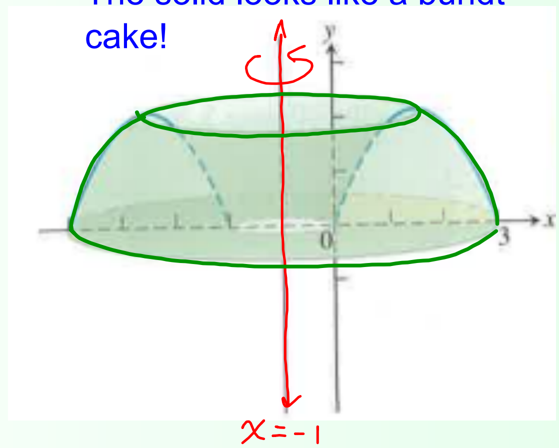
6.3 The Cylindrical Shell Method

Comparison of disc and shell methods

Cylindrical Shells:

The solid looks like a bundt cake!

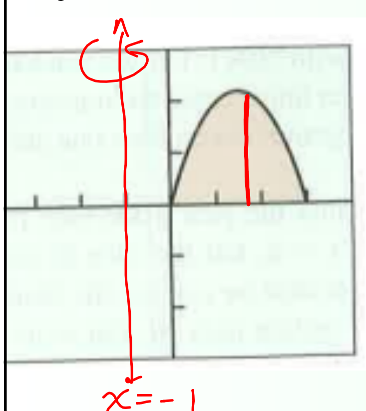


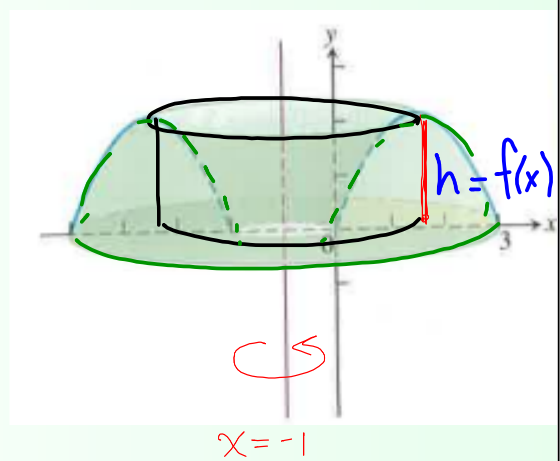


$$\pi\int (R^2 - r^2)\,dy$$

* We need to get R & r in terms of y to use washers revolving around a vertical line.

Cylindrical Shells:





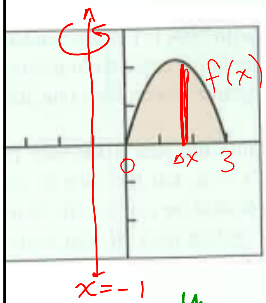
$$V = \int A(x)\,dx$$

A(x) = area of rectangles!

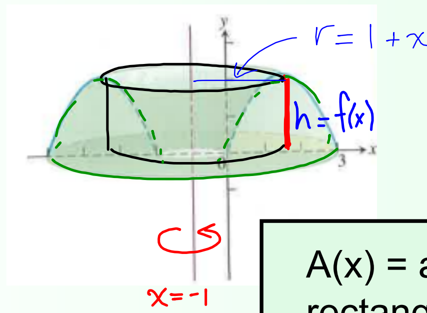
h=f(x)

$2\pi r$

Cylindrical Shells:



y



A(x) = area of rectangles!

h=f(x)

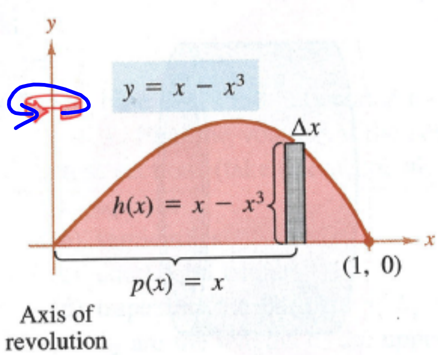
2π(1 + x)

$$V = \int A(x)\,dx$$

$$V = \int_0^3 2\pi(1+x)\cdot f(x)\,dx$$

EXAMPLE 1 Using the shell method to find volume

Find the volume of the solid of revolution formed by revolving the region bounded by $y = x - x^3$ and the x-axis ($0 \le x \le 1$) about the y-axis.



$$\int A(x)\,dx$$

rectangle area

width · height

$$V = \int_0^1 (2\pi x)(x - x^3)\,dx$$

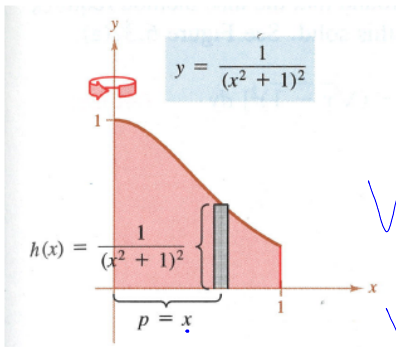
$$= 2\pi \int_0^1 (x^2 - x^4)\,dx$$

EXAMPLE 2 Using the shell method to find volume

Find the volume of the solid of revolution formed by revolving the region bounded by

$$y = \frac{1}{(x^2 + 1)^2}$$

and the x-axis $(0 \le x \le 1)$ about the y-axis.



Axis of revolution

rectangle

$(2\pi r)h$

$\int A(x)\,dx$ $f(x)$

$V = \int 2\pi x (x^2+1)^{-2}\,dx$

$V = \pi \int 2x (x^2+1)^{-2}\,dx$

$u = x^2 + 1$

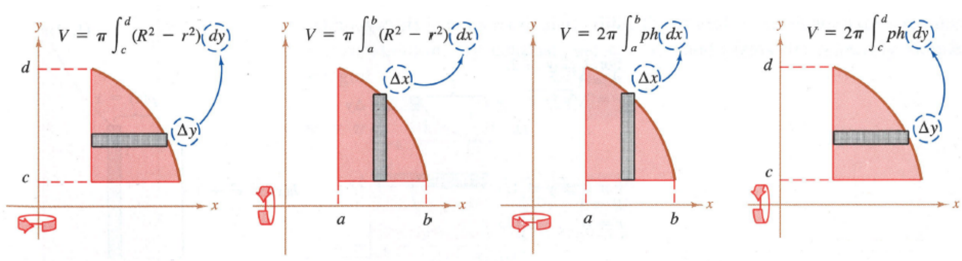
$du = 2x\,dx$

$\pi \int u^{-2}\,du$

Comparison of disc and shell methods ******

The disc and shell methods can be distinguished as follows. For the disc method, the representative rectangle is always *perpendicular* to the axis of revolution, whereas for the shell method, the representative rectangle is always *parallel* to the axis of revolution, as shown in Figure 6.31.

Often, one method is more convenient to use than the other. The following example illustrates a case in which the shell method is preferable.



(a) Vertical axis of revolution Horizontal axis of revolution (b) Vertical axis of revolution Horizontal axis of revolution

IGURE 6.31

EXAMPLE 3 Shell method preferable

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis.

disc + washer

$\pi\int_0^1 1^2\,dy + \pi\int_1^2 \left[1^2-(\sqrt{y-1})^2\right]dy$

$2\pi\int_0^1 x(x^2+1)\,dx$

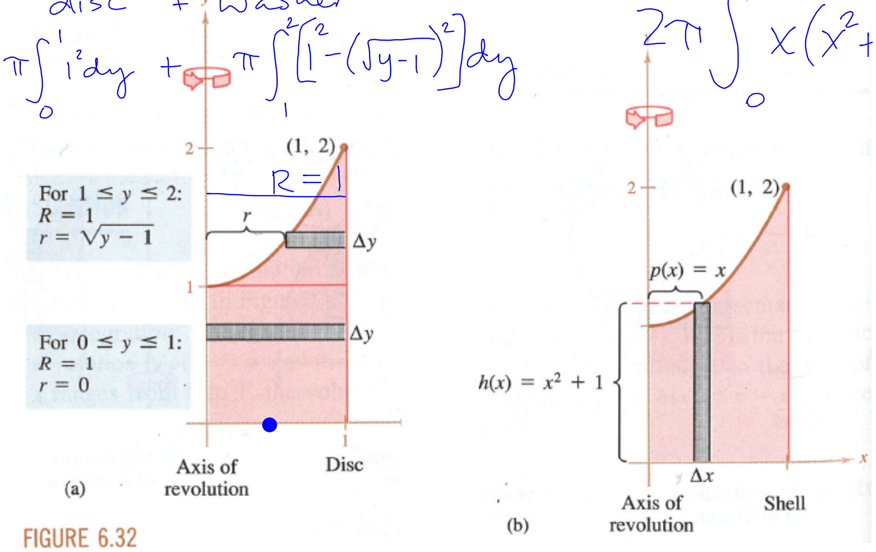


FIGURE 6.32

EXAMPLE 5 Shell method necessary

WHY?

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$, as shown in Figure 6.35.

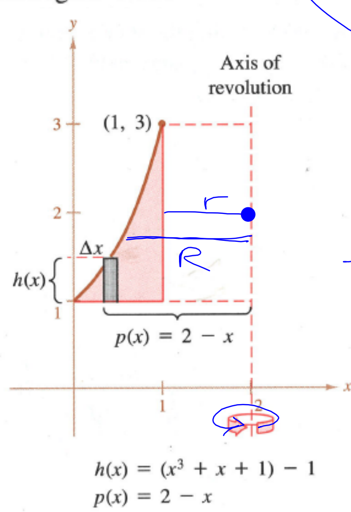


FIGURE 6.35

for washer

$y = x^3 + x + 1$

$x = ?$

to integrate with respect to y.

Shell is easier!

HW: p. 318

1 - 19 odd